CHAPTER 6--PROBABILITY

MATHEMATICAL EXPECTATION AND EXPECTED VALUE

Legend:

N = Notes—copy this in your notebook

A = Activity—I will be explaining and we will do an activity

G = Game—Time to play a Math Game!

C = Classwork, grab a binder, start now and finish it for homework, look up the worksheet/assignment on MHS

N

Mathematical Expectation and Expected Value

Used to determine whether a game that involves betting is fair or not.

If expected value is:

0 = fair game
less than 0 = advantage to house
greater than 0 = advantage to player

formula: $p_1o_1 + p_2o_2 + p_3o_3 + \ldots + p_no_n$ p = probability o = outcome

N

Bet Returned vs. Bet NOT Returned

If you bet \$5 you get to spin the wheel once. When you land on a slice, you will get that amount of money.

Calculate the mathematical expectation if:

- your bet is not returned to you
- your bet is returned to you

Which of the two situations is more realistic for a casino?

N

	Bet NOT Returned	
outcome	**True Value** $(outcome - bet)$	**Probability**
\$0	$\$0 - \$5 = \$ - 5$	$\frac{8}{12} = 0.6667$
\$5	$\$5 - \$5 = \$0$	$\frac{1}{12} = 0.08333$
\$10	$\$10 - \$5 = \$5$	$\frac{1}{12} = 0.08333$
\$15	$\$15 - \$5 = \$10$	$\frac{1}{12} = 0.08333$
\$20	$\$20 - \$5 = \$15$	$\frac{1}{12} = 0.08333$

N

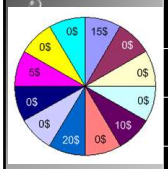
$$M.E. = p_1 \bullet o_1 + p_2 \bullet o_2 + p_3 \bullet o_3 + p_4 \bullet o_4 + p_5 \bullet o_5$$

$$M.E. = (0.6667 \bullet -5) + (0.08333 \bullet 0) + (0.08333 \bullet 5) + (0.08333 \bullet 10) + (0.08333 \bullet 15)$$

$$M.E. = (-3.3335) + (0) + (0.41665) + (0.8333) + (1.24995)$$

$$M.E. = (-0.8336)$$

N



	Bet Returned	
outcome	**True Value** $(outcome)$	**Probability**
\$0	\$0	$\frac{8}{12} = 0.6667$
\$5	\$5	$\frac{1}{12} = 0.08333$
\$10	\$10	$\frac{1}{12} = 0.08333$
\$15	\$15	$\frac{1}{12} = 0.08333$
\$20	\$20	$\frac{1}{12} = 0.08333$

N

$$M.E. = p_1 \bullet o_1 + p_2 \bullet o_2 + p_3 \bullet o_3 + p_4 \bullet o_4 + p_5 \bullet o_5$$

$$M.E. = (0.6667 \bullet 0) + (0.08333 \bullet 5) + (0.08333 \bullet 10) + (0.08333 \bullet 15) + (0.08333 \bullet 20)$$

$$M.E. = (0) + (0.41665) + (0.8333) + (1.24995) + (1.6667)$$

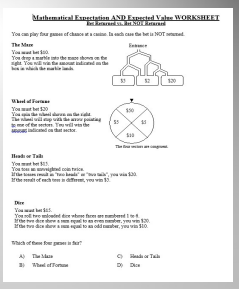
$$M.E. = (4.1666)$$

N

The more realistic situation is when the bet is not returned because "the house always wins". Casinos go into business to make money and they make money when the mathematical expectation is negative.

C

Classwork/ Homework



Mathematical Expectation AND Expected Value WORKSHEET
Bet Returned vs. Bet NOT Returned

You can play four games of chance at a casino. In each case the bet is NOT returned.

The Maze
You must bet $10.
You drop a marble into the maze shown on the right. You will win the amount indicated on the box in which the marble lands.

Entrance
$5 $2 $20

Wheel of Fortune
You must bet $20
You spin the wheel shown on the right.
The wheel will stop with the arrow pointing to one of the sectors. You will win the amount indicated in that sector.

$50 $5 $10

The four sectors are congruent.

Heads or Tails
You must bet $15.
You toss an unweighted coin twice.
If the tosses result in "two heads" or "two tails", you win $30.
If the result of each toss is different, you win $5.

Dice
You must bet $15.
You roll two unloaded dice whose faces are numbered 1 to 6.
If the two dice show a sum equal to an even number, you win $20.
If the two dice show a sum equal to an odd number, you win $10.

Which of these four games is fair?

A) The Maze
B) Wheel of Fortune
C) Heads or Tails
D) Dice

N

Geometric Probability and M. E.

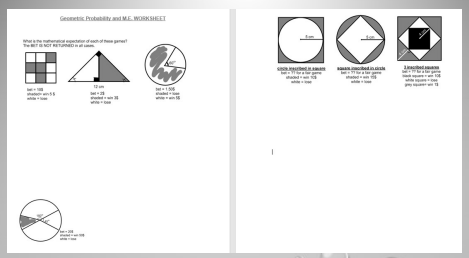
1-D: $\frac{\textbf{length } of\ WANT}{\textbf{length } of\ EVERYTHING}$

2-D: $\frac{\textbf{area } of\ WANT}{\textbf{area } of\ EVERYTHING}$

3-D: $\frac{\textbf{volume } of\ WANT}{\textbf{volume } of\ EVERYTHING}$

C

Classwork/Homework



Geometric Probability and M.E. WORKSHEET

What is the mathematical expectation of each of these games?
The BET IS NOT RETURNED in all cases.

N

Making the Game Fair

Steps:

1. Make the bet = x (we need to figure it out)
2. Create the Table (Outcome/True Value/Probability)
3. Under the column for TRUE VALUE subtract x from the outcome
4. Set up the Equation
   Math.Exp. = $p_1o_1 + p_2o_2 + p_3o_3 \ldots$
5. Make the Math.Exp. = 0

N

Ex 1:

Determine the amount of the bet to make the following game fair.

Roll a six-sided die.

If you roll a 1 you win $10
If you roll a 6 you win $5
Any thing else you lose your bet
NOTE: The bet is NOT returned

N

OUTCOME	TRUE VALUE	PROBABILTY
1 $10	$10 - x$	$1/6 = 0.17$
6 $5	$5 - x$	$1/6 = 0.17$
2,3,4,5 LOSE BET $0	$0 - x$	$4/6 = 0.67$

M.E. = $(\frac{1}{6})(10-x) + (\frac{1}{6})(5-x) + (\frac{4}{6})(-x)$

M.E. = $(\frac{10-x}{6})+(\frac{5-x}{6})+(\frac{-4x}{6})$

$0 = (\frac{10-x+5-x-}{6})$

$0 = (\frac{15-6x}{6})$

$0 = 2.5 - x$

$x = 2.5$

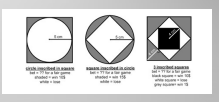
The bet would have to be $2.50 for this game to be fair

C

Classwork/Homework

Green Binders
Chapter 6—Probability
p. 382 #7-10

2nd page of Geometric Probability Worksheet



N

Identifying MHS Questions

MHS presents 3 types of questions which you must learn to identify in order to be able to solve:

- No bet mentioned, just use $p_1o_1 + p_2o_2 + p_3o_3 \ldots$
- Implies bet not returned.
- True Value is assigned in the question…no calculations necessary.

C

Classwork/Homework

Green Binders
Chapter 6—Probability
p. 381 #1-6

Online Assignment
(due in one week)

N

Mathematical Expectation and non-game situations

Kind of like calculating weighted mean

Step 1: create a table Out/T.V./Prob
Step 2: complete it
NOTE: sometimes it will be "bet returned", and sometimes it will be "bet NOT returned", but it won't say so directly and you have to use the context to determine which is appropriate.
Step 3: calculate E.V. or M.E.

Sample situations:

- expected weight
- expected volume
- expected height
- expected cost
- expected profit
- expected time

C

Classwork/Homework

Mathematical Expectation
(non-game situations)
Worksheet