Chapter 4—Geometry
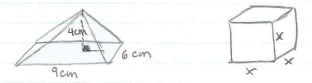EQUIVALENT SOLIDS

N

$$Vol = Vol$$

- Two solids are equivalent if they have the same volumes.
- If you know that two solids are equivalent, you can find a missing dimension from one of the solids. All you have to do is let the two volumes equal each other and solve for the missing dimension.

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VOLUME = VOLUME

Ex.1 A prism with a height of 4cm has a rectangular base with dimensions 6cm by 9cm. What is the height of an equivalent cube?
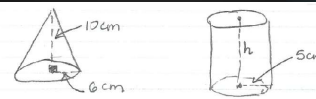

N

$$VOL_{prism} = VOL_{cube}$$
$$A_{Base} \cdot h = l \cdot w \cdot h$$
$$(l \cdot w) \cdot h = l \cdot w \cdot h$$
$$6 \cdot 9 \cdot 4 = x \cdot x \cdot x$$
$$\sqrt[3]{216} = \sqrt[3]{x^3}$$
$$6 = x$$

The height of the cube is 6cm.

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Ex.2 A cone and a cylinder are equivalent. The radius and the height of the cone measure 6cm and 10cm respectively. What is the height of the cylinder if its radius measures 5cm.
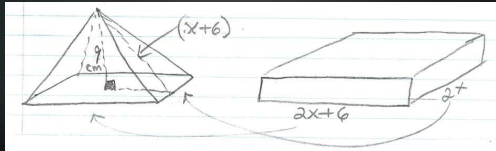

N

$$VOL_{cone} = VOL_{cyl}$$
$$\frac{A_B \cdot h}{3} = A_B \cdot h$$
$$\frac{\pi r^2 \cdot h}{3} = \pi r^2 \cdot h$$
$$\frac{\pi 6^2 \cdot 10}{3} = \pi \cdot 5^2 \cdot h$$
$$\frac{\pi \cdot 36 \cdot 10}{3} = \pi \cdot 25 \cdot h$$
$$\frac{120\pi}{25\pi} = \frac{25\pi \cdot h}{25\pi}$$
$$4.8 = h$$

EX 3

A solid is formed by a pyramid mounted on top of a rectangular prism. The pyramid and the prism are equivalent. The height of the pyramid is 9 units, and the slant height is $(x+6)$. The dimensions of the prism base are $2x$ and $(2x+6)$. What is the numerical value of this solid's total volume?

Step 1

$a^2 + b^2 = c^2$

$9^2 + (x+3)^2 = (x+6)^2$

$81 + x^2 + 6x + 9 = x^2 + 12x + 36$

$(x+3)(x+3)$
$x^2 + 3x + 3x + 9$
$x^2 + 6x + 9$

$(x+6)(x+6)$
$= x^2 + 6x + 6x + 36$
$= x^2 + 12x + 36$

$81 + x^2 + 6x + 9 = x^2 + 12x + 36$

$x^2 - x^2 + 6x - 12x = 36 - 9 - 81$

$\dfrac{-6x}{-6} = \dfrac{-54}{-6}$

$x = 9\ cm$

Step 2

$VOL_{PYR} = VOL_{PRISM}$

$\dfrac{l \cdot w \cdot h}{3} = l \cdot w \cdot h$

$\dfrac{(2x+6) \cdot 2x \cdot x}{3} = (2x+6)(2x)(h)$

x=9 from Step 1

$\dfrac{[2(9)+6] \cdot 2(9) \cdot 9}{3} = [2(9)+6][2(9)] \cdot h$

$\dfrac{(18+6) \cdot 18 \cdot 9}{3} = (18+6)(18) \cdot h$

$\dfrac{24 \cdot 18 \cdot 9}{3} = 24 \cdot 18 \cdot h$

$\dfrac{1296}{432} = \dfrac{432 h}{432}$

$3\ cm = h$

Step 3

$TOT\ VOL = VOL_{PYR} + VOL_{PRISM}$

$= \dfrac{l \cdot w \cdot h}{3} + l \cdot w \cdot h$
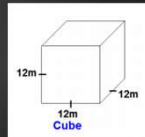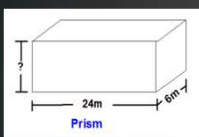
$= \dfrac{24 \cdot 18 \cdot 9}{3} + 24 \cdot 18 \cdot 3$

$= 1296 + 1296$

$= 2592\ cm^3$

Example 4:

▶ Find the missing height of the prism, knowing that it is equivalent to the cube.

Prism

Cube

Step 1: Let the volume of the cube equal the volume of the prism and solve for the height.

Vol. Cube $=$ Vol. Prism

$(l \bullet w \bullet h)_{cube} = (l \bullet w \bullet h)_{prism}$

$(12m)^3 = 24m \bullet 6m \bullet h$

$1728\ m^3 = 144\ m^2 \bullet h$
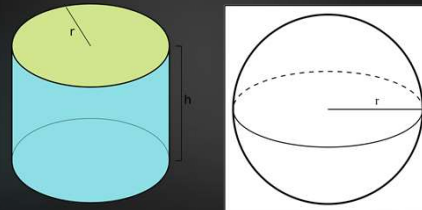
$\dfrac{1728\ m^3}{144\ m^2} = h$

$12\ m = h$

The prism is 12 m high.

N **Example 5:**

► A cylinder and a sphere have the same radius of 6m and are equivalent. What is the height of the cylinder?

N Step 1:

► Draw a diagram.

N Step 2: Let the volume of the cylinder equal the volume of the sphere and solve for h, the height of the cylinder.

Volume of Cylinder = Volume of Sphere

$$(\pi)(r)^2(h) = \left(\frac{4}{3}\right)(\pi)(r)^3$$

$$(\pi)(6m)^2(h) = \left(\frac{4}{3}\right)(\pi)(6m)^3$$

$$(36\pi m^2)(h) = \frac{(4)(\pi)(216m^3)}{3}$$

$$h = \frac{864\pi m^3}{(3)(36\pi m^2)}$$

$$h = \frac{864\pi m^3}{108\pi m^2}$$

$h = 8m$ is the height

C Classwork/homework

► MHS Worksheet "Chapter Equivalent Solids p.260 # 1-10

► Online Assignment Equivalent Solids